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Implications for the formation of star clusters from extra-galactic star-formation rates

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ABSTRACT

Observations indicate that young massive star clusters in spiral and dwarf galaxies follow a relation between luminosity of the brightest young cluster and the star-formation rate (SFR) of the host galaxy, in the sense that higher SFRs lead to the formation of brighter clusters. Assuming that the empirical relation between maximum cluster luminosity and SFR reflects an underlying similar relation between maximum cluster mass ($M_{\rm ecl,max}$) and SFR, we compare the resulting $SFR(M_{\rm ecl,max})$ relation with different theoretical models. The empirical correlation is found to suggest that individual star clusters form on a free-fall time-scale with their pre-cluster molecular-cloud-core radii typically being a few pc independent of mass. The cloud cores contract by factors of 5 to 10 while building-up the embedded cluster. A theoretical $SFR(M_{\rm ecl,max})$ relation in very good agreement with the empirical correlation is obtained if the cluster mass function of a young population has a Salpeter exponent $\beta \approx 2.35$ and if this cluster population forms within a characteristic time-scale of a few-10 Myr. This short time-scale can be understood if the inter-stellar medium is pressurised thus precipitating rapid local fragmentation and collapse on a galactic scale. Such triggered star formation on a galactic scale is observed to occur in interacting galaxies. With a global SFR of $3-5\,M_\odot$/yr the Milky Way appears to lie on the empirical $SFR(M_{\rm ecl,max})$ relation, given the recent detections of very young clusters with masses near $10^5\,M_\odot$ in the Galactic disk. The observed properties of the stellar population of very massive young clusters suggests that there may exist a fundamental maximum cluster mass, $10^6 < M_{\rm ecl,max*}/M_\odot < 10^7$.

Key words: stars: formation – open clusters and associations – galaxies: star clusters – galaxies: interactions – galaxies: star-burst – galaxies: evolution

1 INTRODUCTION

In a series of publications Larsen (Larsen 2000, 2001, 2002) and Larsen & Richtler (2000) examined star cluster populations of 37 spiral and dwarf galaxies and compared the derived properties with overall attributes of the host galaxy. For this work they used archive HST data, own observations and literature data. They showed that cluster luminosity functions (LFs) are very similar for a variety of galaxies. They also found that the V-band luminosity of the brightest cluster, $M_{\rm V}$, correlates with the global star-formation rate, SFR, but it is unclear if this correlation is of physical or statistical nature. According to the statistical explanation there is a larger probability of sampling more luminous clusters from a universal cluster LF when the SFR is higher (Larsen 2002; Billett, Hunter & Elmegreen 2002).

Larsen (2001) concluded that all types of star clusters form according to a similar formation process which operates with different masses. Smaller clusters dissolve fast through dynamical effects (gas expulsion, stellar-dynamical heating, galactic tidal field) and only massive clusters survive for a significant fraction of a Hubble time (Vesperini 1998; Fall & Zhang 2001; Baumgardt & Makino 2003). The notion is that virtually all stars form in clusters (Kroupa & Boily 2002; Lada & Lada 2003), and that a star-formation "epoch" produces a population of clusters ranging from about $5\,M_\odot$ (Taurus-Auriga-like pre-main sequence stellar groups) up to the heaviest star cluster which may have a mass approaching $10^6\ M_\odot$. The time-scale over

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which such a cluster population emerges within a galaxy defines its momentary SFR.

The aim of this contribution is to investigate if the empirical $M_V(SFR)$ relation may be understood to be a result of physical processes. In § 2 the observational data concerning the correlation between the SFR and M_V of the brightest star cluster are presented, and the empirical and physical models describing this correlation are elaborated in § 3. § 4 contains the discussion and conclusion.

2 THE OBSERVATIONAL DATA

Based on various observational results Larsen (2002) concludes that star clusters form under the same basic physical processes, and that the so-called super-clusters are just the young and massive upper end of the distribution. We firstly derive from this observational material a correlation between the absolute magnitude of the brightest cluster and the star-formation rate of the host galaxy.

Including in Fig.1 all data-points presented by Larsen (2001, 2002) the following equation (1) emerges from a 2-dimensional linear least square fit

$$\mathrm{M}_V = -1.93(\pm 0.06) \times \log SFR - 12.55(\pm 0.07) \quad (1)$$

with a reduced $\chi^2_{\rm red}$ of about 17. Excluding four points (A, B, C and D, see Larsen 2002) that lie far above this first fit leads to

$$\mathrm{M}_V = -1.87(\pm 0.06) \times \log SFR - 12.14(\pm 0.07) \quad (2)$$

with a reduced $\chi^2_{\rm red}$ of about 6. Both fits are shown in Fig. 1. For the magnitude M_V the *formal* error is based on photon statistics, and is always very small (especially since these are the brightest clusters in the galaxies), usually 0.01 mag or less. Most of the errors are systematic, due to uncertain aperture corrections, contamination within the photometric aperture by other objects and are typically 0.1 mag. The SFRs are derived from IR-fluxes puplished in the IRAS catalog which lists typical errors of 15%. However, a major source of uncertainty in the derived SFRs lies in the FIR luminosity vs SFR calibration, for which Buat & Xu (1996) quote a typical error of +100%/-40%.

Inverting eq. 2 reveals,

$$\log SFR = -0.54(\pm 0.02) \times \mathrm{M}_V - 6.51(\pm 0.26), \quad (3)$$

while a fit to the inverted data (SFR vs M_V) gives

$$\log SFR = -0.54(\pm 0.02) \times \mathrm{M}_V - 6.51(\pm 0.19), \quad (4)$$

with a reduced $\chi^2_{\rm red}$ of about 6. Both eqs. 3 and 4 lead to essentially the same result thus nicely demonstrating its robustness.

The exclusion of A, B, C and D is motivated by three of them being clusters in very sparse cluster systems in dwarf galaxies (DDO 165, NGC 1705 and NGC 1569) dominated by a single brightest member. Therefore the *present* SFR does not describe the rate during the birth of these clusters. It has dropped to the shown values as no further (massive) clusters are seen to be forming. This can be understood as a general trend of aging after a star-formation epoch. The underlying (observed) SFR has dropped while the clusters retain an approximately constant luminosity for the first few million years (Table 1). The clusters therefore appear on

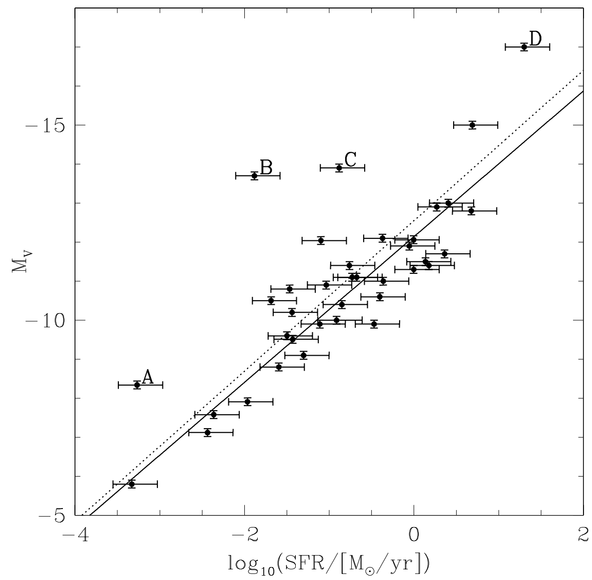

Figure 1. Observational data from Larsen (2002) for absolute magnitude of the brightest cluster versus global star-formation rate for 37 disk and dwarf galaxies. The solid line illustrates a linear regression fit to the data excluding the four data points (A, DDO 165), (B, NGC 1705), (C, NGC 1569) and (D, NGC 7252) while the dotted line includes all points.

the left in this diagram (Fig. 1) in dwarf galaxies in which SF proceeds in bursts. The cluster in NGC 7252 was excluded because this galaxy is a several 10^8 yr old merger in which the SFR was presumably much higher when most of its clusters formed and in which the brightest "cluster" is probably an unresolved or already merged star-cluster complex (Fellhauer & Kroupa 2002a), and thus not a true single cluster.

3 THE MODELS

3.1 Empirical model

From the second fit to the observations (eq. 2) we derive an empirical model for the dependence of the mass of the heaviest cluster on the underlying star-formation rate of the host galaxy. With the use of the mass-to-light-ratio, $k_{\rm ML}$, the magnitude ($\mathrm{M_V}$) can be converted to a mass ($M_{\rm ecl,max}$),

$$\mathrm{M_V} = 4.79 - 2.5 \cdot \log \frac{M_{\rm ecl,max}}{k_{\rm ML}}, \quad (5)$$

where $M_{\rm ecl,max}$ is the stellar mass in the cluster. The mass-to-light ratios in Table 1 are derived from Smith & Gallagher (2001, fig. 7). The age spread between 6.0 and 8.0 (in logarithmic units) is used to estimate the mass errors for the individual clusters in the Larsen data set plotted in Fig. 3 below.

Substituting eq. 2 in 5 gives,

$$M_{\rm ecl,max} = k_{\rm ML} \cdot SFR^{0.75(\pm 0.03)} \cdot 10^{6.77(\pm 0.02)} \quad (6)$$

and eq. 5 in 4,

$$SFR = \left(\frac{M_{\rm ecl,max}}{k_{\rm ML}}\right)^{1.34(\pm 0.04)} \cdot 10^{-9.07(\pm 0.28)} \quad (7)$$

Table 1. Mass-to-light ratios.

k_{ML}	log age [yr]
0.0144	$6.0 < t < 6.8$
0.0092	7.0
0.1456	8.0

The question whether the brightest cluster observed is always the heaviest is non-trivial to answer because for example a less-massive but somewhat younger cluster may appear brighter than an older more massive cluster, because the stellar population fades with age. This does not always hold true for the very youngest phases, where the clusters may briefly brighten somewhat due to the appearance of red supergiant stars (Table 1). We therefore explore this problem with a rather simple model. Using three different cluster formation rates (CFR; linear decreasing, linear increasing and constant) a number of clusters is formed per time-step (1 Myr). Taking a power-law CMF with an exponent $\beta = 2$ (eq. 11 in § 3.4.1 below) cluster masses are allocated randomly by a Monte-Carlo method. These clusters are then evolved using time dependent mass-to-light ratios derived from a Starburst99 simulation (Leitherer et al. 1999) for a Salpeter IMF ($\alpha = 2.35$) from $0.18\,M_\odot$ to $120\,M_\odot$ for a $M_{\mathrm{ecl}} = 10^6\,M_\odot$ cluster over 1 Gyr. The lower mass boundary is chosen in order to have the same mass in stars in the cluster with the Salpeter IMF as in a universal Kroupa IMF (Kroupa 2001). The evolution of M_V of a $M_{\mathrm{ecl}} = 10^6\,M_\odot$ and a $M_{\mathrm{ecl}} = 5 \times 10^5\,M_\odot$ cluster is shown in Fig. 2.

For the whole Monte-Carlo Simulation the heaviest cluster is also the brightest for about 95% of the time and for all three cases of the CFR over the first 500 Myr. Therefore we estimate an uncertainty of about 5% on our assertion that the brightest cluster in a population is also the most massive one. This uncertainty can be neglected relatively to the larger uncertainties in the cluster ages and therefore in the mass-to-light ratios.

Larsen (2002) points out that a relation between the luminosity of the brightest cluster, M_{V}, and the total SFR arrived at by random sampling from a power-law LF, given an area-normalised star formation rate Σ_{SFR} and total galaxy size, reproduces the observed correlation. The aim of this contribution is to investigate if the correlation may be the result of physical processes. In essence, the observed correlation is expected because in order to form a massive cluster in a similar time-span a higher SFR is needed than for a low-mass cluster. In order to probe the physical background of the empirical relation (eqs. 6 and 7) we calculate a number of different models in § 3.2 to 3.4.

3.2 Local data model

In Fig. 3 the data for individual clusters in the Milky Way (Taurus-Auriga, Orion Nebula cluster) and in the LMC (R136, the core of the 30 Doradus region) are compared with the extragalactic cluster-system data. The data points (crosses) were calculated by dividing a mass estimate for each cluster by a formation time of 1 My. This is a typical formation time-scale as deduced from the ages of the stars in Taurus–Auriga, the Orion Nebula cluster and in R136.

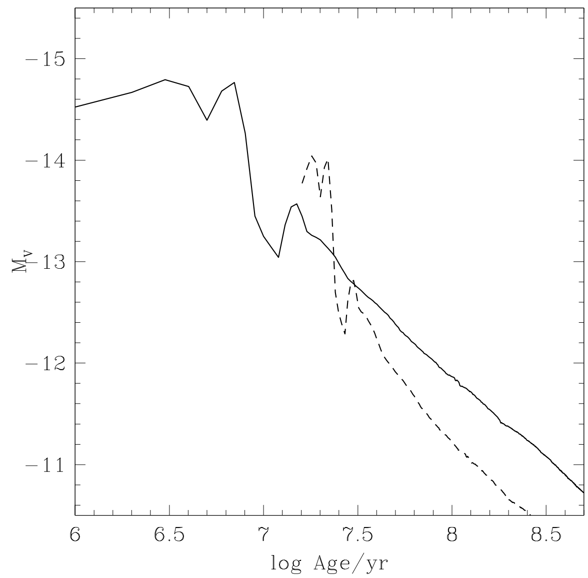

Figure 2. Temporal evolution of the visual magnitude M_V of a $10^6\,M_\odot$ cluster (solid line) and a $5 \times 10^5\,M_\odot$ cluster (dashed line) over 500 Myr with mass-to-light ratios derived from a Starburst99 simulation (Leitherer et al. 1999). In this example the $5 \times 10^5\,M_\odot$ cluster forms when the $10^6\,M_\odot$ cluster is 1.5×10^7 years old and appears brighter than the more massive cluster for about 10 Myr.

The error for the mass scale is constructed from different assumptions in the literature about the number of stars in the cluster and with the use of different mean masses as they vary in dependence of the used IMF and the maximal possible stellar mass for the particular cluster. We thus have upper and lower bounds on the cluster masses. By dividing the upper mass over a formation time of 0.5 Myr and the lower mass over 2 Myr the corresponding errors for the SFR are obtained. The data for these assumptions are taken from Briceño et al. (2002), Kroupa (2001), Hartmann (2002), Massey & Hunter (1998), Palla & Stahler (2002) and Selman et al. (1999).

Fig. 3 demonstrates that this simplest description already leads to reasonable agreement with the observational data. That the local individual cluster data are offset to lower SFRs from the extragalactic data can be understood as being due to the observations measuring the SFR for entire star-cluster populations rather than for individual clusters and/or the formation time-scale to vary with cluster mass. In § 3.3 the star-cluster formation time scale (set here to be 1 Myr) is allowed to be the mass-dependent free-fall time-scale.

3.3 Free-fall model

Here the time scale for the formation of an individual star cluster is the free-fall-time t_{ff} for a pre-cluster molecular cloud core with radius R. This model is motivated by the insight by Elmegreen (2000) that star formation occurs on virtually every level, from galactic scales over clusters to stars themselves, within one or two crossing times. The SFR needed to build-up one (e.g. the most massive) cluster in a free-fall time is

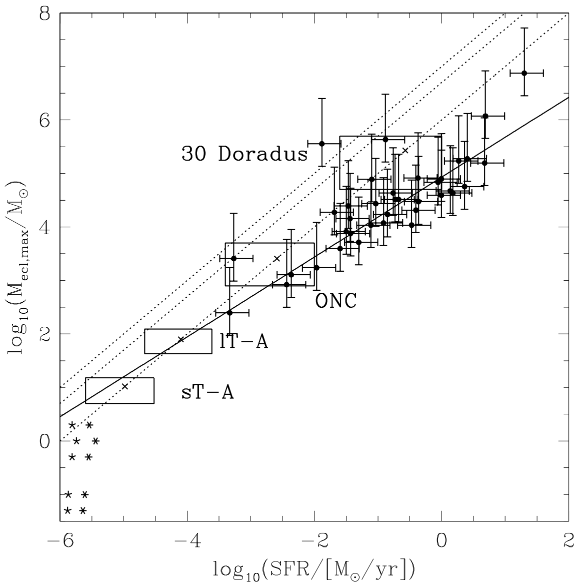

Figure 3. Maximum cluster mass versus global star-formation rate (SFR), both in logarithmic units. Filled dots are observations by Larsen with error estimates (see § 3.1 for details) and the linear regression fit is the solid line (eq. 6). The crosses are different Galactic and extra-galactic clusters with the SFR for each one obtained by simply dividing the mass by a formation time of one Myr. The construction of the error boxes is explained in § 3.2. The individual clusters are: sT-A: small sub-clumps in Taurus-Auriga, lT-A: the whole Taurus-Auriga star-forming region, ONC: Orion Nebula cluster, 30 Doradus: the R136 cluster in the 30 Doradus star-forming region in the LMC. The dotted lines show $M_{\rm ecl,max}(SFR)$ relations assuming a 1 (bottom), 5 and 10 (top) Myr formation time for individual clusters (ie. not cluster-systems). The two types of asterisks are single stars with final main-sequence masses of 0.05, 0.1, .0.5, 1.0 and 2.0 $M_\odot$ (from bottom to top) after accretion of 90 pert cent (on the left) and 99 per cent (on the right) of their mass, from Wuchterl & Tscharnuter (2003).

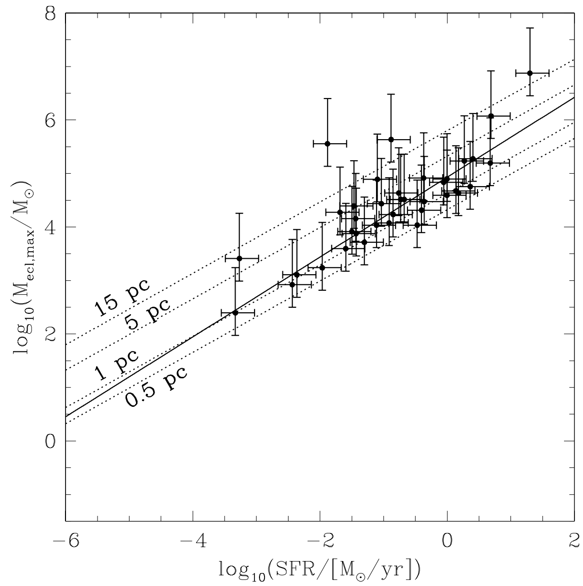

Figure 4. As Fig. 3 but the dotted lines show the model SFR needed to build a single cluster within one free-fall time (eq. 10) for $R = 0.5$, 1, 5 and 15 pc.

$$SFR = \frac{M_{\rm ecl,max}}{t_{\rm ff}}. \tag{8}$$

For the free-fall-time we take the dynamical time-scale (for simliar considerations see e.g. Elmegreen 2000),

$$t_{\rm ff} \approx \sqrt{\frac{R^3}{G \cdot M_{\rm st+g}}}, \tag{9}$$

where $M_{\rm st+g}$ is the total mass of the embedded cluster including gas and stars. With a star-formation efficiency of 33% (Lada & Lada 2003; Kroupa, Aarseth & Hurley 2001) we have $M_{\rm st+g} = 3M_{\rm ecl,max}$. The combination of 8 and 9 leads to

$$SFR = \sqrt{3}\ M_{\rm ecl,max}^{3/2} \frac{\sqrt{\rm G}}{R^{3/2}}, \tag{10}$$

with $\mathrm{G} = 4.485 \cdot 10^{-3}\ \mathrm{pc}^3 M_\odot^{-1}\ \mathrm{Myr}^{-2}$ being the gravitational constant. Fig. 4 shows this relation for $R = 0.5$, 1, 5 and 15 pc.

Thus a simple model based on the SFR required to form one cluster in a free-fall time leads to a $M_{\rm ecl,max}(SFR)$ relation in good agreement with the empirical relation provided the pre-cluster cloud cores have radii of about 5 pc nearly independently of their mass, because the correct relation ought to lie leftward of the empirical data in Fig. 4 since the empirical SFRs are for entire cluster populations.

The groups of pre-main sequence stars in Taurus-Auriga (a few $M_\odot$) have radii of about 0.5 pc (Gomez et al. 1993). The about 1 Myr old Orion Nebula cluster (a few 1000 $M_\odot$) has a radius of about 1 pc (Hillenbrand & Hartmann 1998) but it is most probably expanding owing to gas expulsion (Kroupa, Aarseth & Hurley 2001). The $1-3$ Myr old R136 in the LMC ($\approx 10^{4-5}\ M_\odot$), which today is seen to have a radius of a few pc (Brandl et al. 1996), is most likely also in a post-gas-expulsion expansion phase. Also Maíz-Apellániz (2001) notes from his sample of 27 massive ($\gtrsim 3 \times 10^4\ M_\odot$) and young ($<$ 20 Myr) clusters that those younger than about 7 Myr have radii of about 1 pc only. *Very young, still-embedded clusters appear to be very compact with radii of 0.5–1 pc, and the results from Fig. 4 can be taken to mean that they form in a free-fall time if the pre-cluster cloud cores have radii of about 5 pc at the onset of collapse.* The build-up of the stellar population would proceed while the density of the cloud core increases by a factor of about 5^3 to 10^3, when the star-formation rate in the embedded cluster probably peaks and declines rapidly thereafter as a result of gas evacuation from accumulated outflows and/or the formation of the massive stars that photo-ionise the cloud core (Matzner & McKee 2000; Tan & McKee 2002).

3.4 Total-mass model

The above free-fall model quantifies the theoretical relation for the case that the measurements only capture star-formation in the most massive clusters in a galaxy. This can be considered to be a lower-bound on the SFR. An upper bound is given by the rate with which all stars are being formed, which means the total mass being converted to stars in a given time interval. This total mass is the mass in the star-cluster population and is the subject of this subsection,

which begins by assuming there exists no fundamental maximum star-cluster mass, followed by an analysis in which a physical maximum cluster mass, $M_{\mathrm{ecl,max}*}$, is incorporated.

3.4.1 Without a physical maximum cluster mass

The aim is to estimate the SFR required to build a complete young star-cluster population in one star-formation epoch such that it is populated fully with masses ranging up to $M_{\mathrm{ecl,max}}$. Observational surveys suggest the embedded-cluster mass function (CMF) is a power-law,

$$\xi_{\mathrm{ecl}}(M_{\mathrm{ecl}}) = k_{\mathrm{ecl}} \cdot \left(\frac{M_{\mathrm{ecl}}}{M_{\mathrm{ecl,max}}}\right)^{-\beta}, \tag{11}$$

with $1.5 \lesssim \beta \lesssim 2.5$ (Elmegreen & Efremov 1997; Kroupa 2002; Kroupa & Boily 2002; Lada & Lada 2003; Kroupa & Weidner 2003, and references therein). For the total mass of a population of young stellar clusters,

$$\begin{aligned} M_{\mathrm{tot}} &= \int_{M_{\mathrm{ecl,min}}}^{M_{\mathrm{ecl,max}}} M_{\mathrm{ecl}} \cdot \xi_{\mathrm{ecl}}(M_{\mathrm{ecl}})\, dM_{\mathrm{ecl}} \\ &= k_{\mathrm{ecl}} \cdot M_{\mathrm{ecl,max}}^{\beta} \cdot \int_{M_{\mathrm{ecl,min}}}^{M_{\mathrm{ecl,max}}} M_{\mathrm{ecl}}^{1-\beta}\, dM_{\mathrm{ecl}}, \end{aligned} \tag{12}$$

where $M_{\mathrm{ecl,max}}$ is the mass of the heaviest cluster in the population. The normalisation constant k_{ecl} is determined by stating that $M_{\mathrm{ecl,max}}$ is the single most massive cluster,

$$\begin{aligned} 1 &= \int_{M_{\mathrm{ecl,max}}}^{\infty} \xi_{\mathrm{ecl}}(M_{\mathrm{ecl}})\, dM_{\mathrm{ecl}} \\ &= k_{\mathrm{ecl}} \cdot M_{\mathrm{ecl,max}}^{\beta} \cdot \int_{M_{\mathrm{ecl,max}}}^{\infty} M_{\mathrm{ecl}}^{-\beta}\, dM_{\mathrm{ecl}}. \end{aligned} \tag{13}$$

With a CMF power-law index of $\beta = 2$ we get from eq. 13,

$$k_{\mathrm{ecl}} = \frac{1}{M_{\mathrm{ecl,max}}}. \tag{14}$$

Inserting this into eq. 12 (again with $\beta = 2$),

$$M_{\mathrm{tot}} = M_{\mathrm{ecl,max}} \cdot (\ln M_{\mathrm{ecl,max}} - \ln M_{\mathrm{ecl,min}}). \tag{15}$$

$M_{\mathrm{ecl,min}}$ is the minimal cluster mass which we take to be $5\,M_{\odot}$ (a small Taurus-Auriga like group). For arbitrary $\beta \neq 2$ eqs. 14 and 15 change to

$$k_{\mathrm{ecl}} = \frac{\beta - 1}{M_{\mathrm{ecl,max}}} \tag{16}$$

and

$$M_{\mathrm{tot}} = (\beta - 1) \cdot M_{\mathrm{ecl,max}}^{\beta-1} \cdot \left(\frac{M_{\mathrm{ecl,max}}^{2-\beta} - M_{\mathrm{ecl,min}}^{2-\beta}}{2-\beta}\right). \tag{17}$$

The resulting total mass, M_{tot}, as a function of the maximal cluster mass, $M_{\mathrm{ecl,max}}$, is shown in Fig. 5 for different β.

Given a SFR, a fully-populated CMF with total mass M_{tot} is constructed in a time δt,

$$M_{\mathrm{tot}} = SFR \cdot \delta t. \tag{18}$$

Thus, dividing M_{tot} by different ad-hoc formation times, δt, and using different maximal masses, $M_{\mathrm{ecl,max}}$, results in a series of theoretical $M_{\mathrm{ecl,max}}(SFR)$ relations which are shown in Fig. 6. *It thus appears that star-formation epochs with duration $\delta t \approx 10$ Myr suffice for populating complete cluster systems.*

The argumentation can now be inverted to better quantify the time-scale required to build an entire young cluster population in a star-formation epoch with a given SFR. For this purposed we employ the empirical $SFR(M_{\mathrm{ecl,max}})$ relation. For $\beta = 2$,

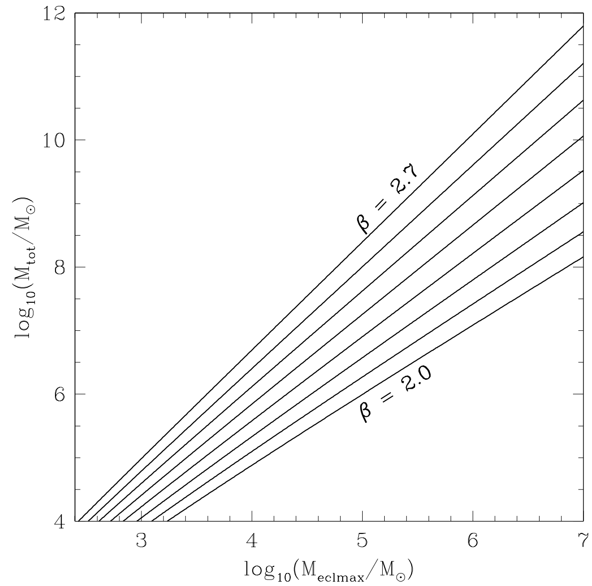

Figure 5. The (logarithmic) total mass of a cluster system, M_{tot}, in dependence of the (logarithmic) maximal cluster mass, $M_{\mathrm{ecl,max}}$, for different CMF power-law indices β (= 2.0 to 2.7, from bottom to top)

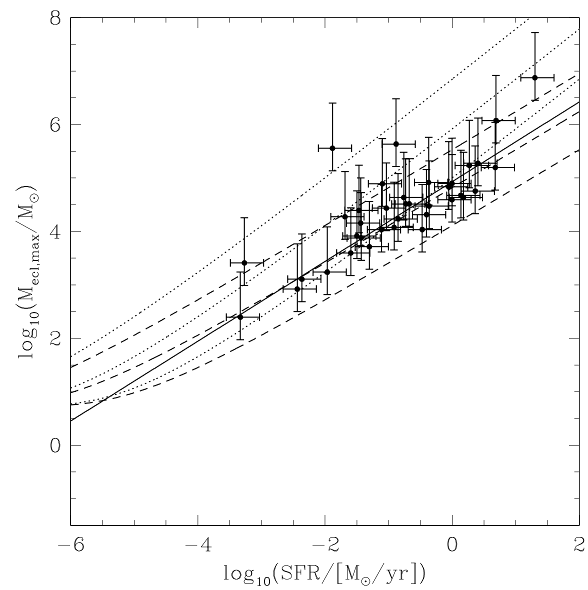

Figure 6. Same as Fig. 3. However, here the theoretical relations (eqs. 15 and 18 or 17 and 18) assume the entire young-cluster population forms in $\delta t = 1$, 10 and 100 Myr (bottom to top). The CMF has $\beta = 2$ (dotted curves) or $\beta = 2.4$ (dashed curves).

$$\delta t = \frac{M_{\mathrm{tot}}}{SFR} = \frac{M_{\mathrm{ecl,max}}}{SFR} \ln\left(\frac{M_{\mathrm{ecl,max}}}{M_{\mathrm{ecl,min}}}\right). \tag{19}$$

Eq. 7 can be re-written,

$$SFR = \left(\frac{M_{\mathrm{ecl,max}}}{k_{\mathrm{ML}}}\right)^{s} 10^{-9.07}, \tag{20}$$

with $s = 1.34$ being the exponent of this empirical $SFR(M_{\rm ecl,max})$ relation. Combining eqs. 19 and 20 we finally arrive at

$$\delta t = M_{\rm ecl,max}^{1-s} \ln\left(\frac{M_{\rm ecl,max}}{M_{\rm ecl,min}}\right) k_{\rm ML}^{s} \cdot 10^{9.07} \text{ [yr]}. \tag{21}$$

For $\beta \neq 2$ we obtain instead,

$$\delta t = \quad (\beta-1)\, M_{\rm ecl,max}^{\beta-1-s} \left(\frac{M_{\rm ecl,max}^{2-\beta} - M_{\rm ecl,min}^{2-\beta}}{2-\beta}\right) \times k_{\rm ML}^{s} \cdot 10^{9.07} \text{ [yr]}. \tag{22}$$

The cluster-system formation time scale, or the duration of the star-formation "epoch", δt, is plotted in Fig. 7 for different $M_{\rm ecl,max}$ – and therefore different $M_{\rm tot}$ – and different CMF slopes β. For $\beta \lesssim 2.4$ a decreasing δt for almost all masses is found which indicates that the formation of the whole cluster system can be very rapid ($\lesssim 10$ Myr).

We have thus found that the empirical $SFR(M_{\rm ecl,max})$ relation implies that more-massive cluster populations need a shorter time to assemble than less massive populations, unless the embedded cluster mass function is a power-law with an index of $\beta \approx 2.35$, strikingly similar to the Salpeter index for stars ($\alpha = 2.35$). In this case the formation time becomes ≈ 10 Myr independent of the maximum cluster mass in the population.

Young populations of star clusters extend to super-star clusters mostly in galaxies that are being perturbed or that are colliding. The physics responsible for this can be sought in the higher pressures in the inter-stellar medium as a result of the squeezed or colliding galactic atmospheres (Elmegreen & Efremov 1997; Bekki & Couch 2003). When this occurs, massive molecular clouds rapidly build-up and collapse locally but distributed throughout the galaxy. If two disk galaxies collide face-on, star-formation occurs synchronised throughout the disks, while edge-on encounters would lead to the star-formation activity propagating through the disks with a velocity of a few-100 pc/Myr (the relative encounter velocity) which amounts to a synchronisation of star-formation activity throughout 10 kpc radii disks to within a few-10 Myr. Just recently Engargiola et al. (2003) found that for M33 the typical life-times of giant molecular clouds (GMC) with masses ranging up to $7 \times 10^5 M_\odot$ are 10 to 20×10^6 yr, indicating a similar formation time for star clusters born form these clouds. Hartmann, Ballesteros-Paredes & Bergin (2001) deduce from solar-neighbourhood clouds that their life-times are also comparable to the ages of the pre-main sequence stars found within them, again suggesting that molecular clouds form rapidly and are immediately dispersed again through the immediate on-set of star-formation.

Notable is that $\beta \approx 2.4$ gives a theoretical $M_{\rm ecl,max}(SFR)$ relation with virtually the same slope as the empirical relation (Fig. 6). *The implication would be that the embedded-CMF is essentially a Salpeter power-law.* Also, in the analysis above we neglected to take into account that once an embedded cluster expels its residual gas it expands and loses typically 1/2 to 2/3 of its stars (Kroupa 2002; Kroupa & Boily 2002). The observed clusters with ages > 10 Myr thus have masses $(0.3-0.5) \times M_{\rm ecl}$. Taking this into account would shift the theoretical relations downwards by at most 0.5 in log-mass which would lead to an increase in δt by a factor of a few.

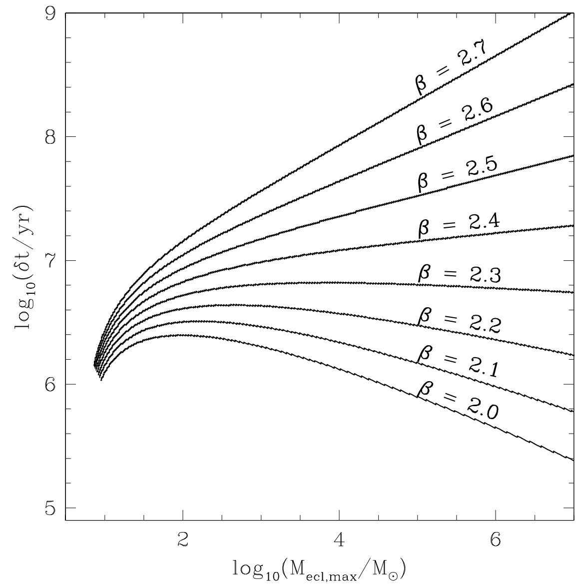

Figure 7. Formation time scale (logarithmic years) of the cluster system (eqs 21 and 22) over maximal cluster mass (in logarithmic units) for different slopes β of the CMF assuming the mass-to-light ratio of young clusters is $k_{\rm ML} = 0.0144$ (Table 1).

3.4.2 With a physical maximal cluster mass

The most massive "clusters" known, e.g. ω Cen (a few $10^6 M_\odot$, Gnedin et al. 2002) or G1 ($\approx 15 \times 10^6 M_\odot$, Meylan et al. 2001) consist of complex stellar populations with different metalicities and ages (Hilker & Richtler 2000). They are therefore not single-metalicity, single-age populations that arise for a truly spatially and temporarily localised star-cluster forming event, but are probably related to dwarf galaxies that formed from a compact population of clusters and with sufficient mass to retain their interstellar medium for substantial times and/or capture field-stellar populations and/or possibly re-accrete gas at a later time to form additional stars (Kroupa 1998; Fellhauer & Kroupa 2002b). A fundamental, or physical maximal star-cluster mass may therefore be postulated to exist on empirical grounds in the range $10^6 \lesssim M_{\rm ecl,max*}/M_\odot \lesssim 10^7$. In the following we explore the implications of such a fundamental maximum cluster mass on the analysis presented in § 3.4.1.

For the following $M_{\rm ecl,max*} = 10^7 M_\odot$ is adopted. Eq. 12 remains unchanged while eq. 13 changes to

$$1 = k_{\rm ecl} \cdot M_{\rm ecl,max}^{\beta} \cdot \int_{M_{\rm ecl,max}}^{M_{\rm ecl,max*}} M_{\rm ecl}^{-\beta}\, dM_{\rm ecl}. \tag{23}$$

This can be evaluated for $\beta \neq 1$

$$k_{\rm ecl} = M_{\rm ecl,max}^{-\beta} \frac{1-\beta}{M_{\rm ecl,max*}^{1-\beta} - M_{\rm ecl,max}^{1-\beta}}. \tag{24}$$

If $\beta = 2$,

$$M_{\rm tot} = -\frac{\ln M_{\rm ecl,max} - \ln M_{\rm ecl,min}}{M_{\rm ecl,max*}^{-1} - M_{\rm ecl,max}^{-1}}, \tag{25}$$

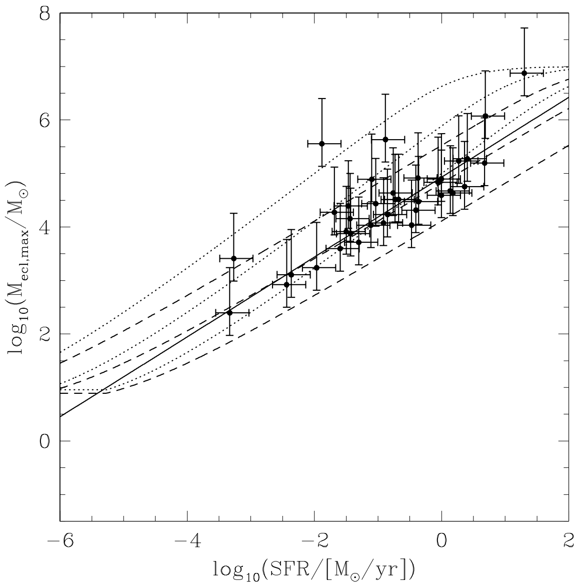

Figure 8. As Fig. 6 but for the case that there exists a fundamental maximum cluster mass $M_{\rm ecl,max*} = 10^7\,M_\odot$.

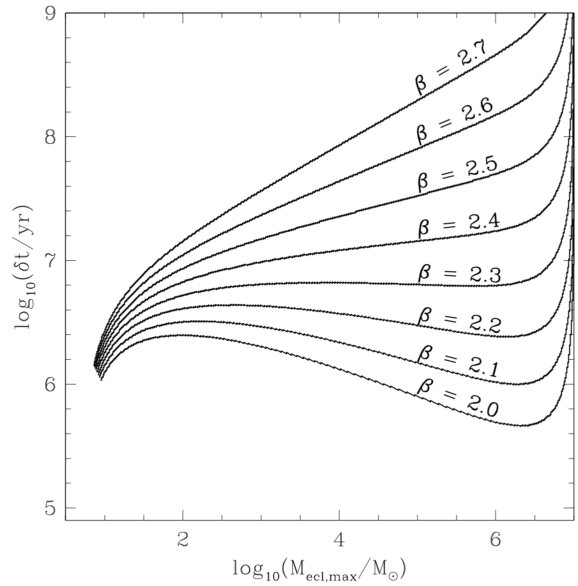

Figure 9. As Fig. 7 but assuming the fundamental cluster mass limit is $M_{\rm ecl,max*} = 10^7\,M_\odot$.

or for $\beta \neq 2$

$$M_{\rm tot} = \frac{1-\beta}{2-\beta}\,\frac{M_{\rm ecl,max}^{2-\beta} - M_{\rm ecl,min}^{2-\beta}}{M_{\rm ecl,max*}^{1-\beta} - M_{\rm ecl,max}^{1-\beta}}. \qquad (26)$$

For a fixed $M_{\rm ecl,max*}$ and a changing $M_{\rm tot}$ the upper mass $M_{\rm ecl,max}$ for each cluster system can now be evaluated. The resulting $SFR(M_{\rm ecl,max})$ models are plotted in Fig. 8 for different formation-times of the entire cluster population. The conclusions of the previous section do not change.

Given the empirical $SFR(M_{\rm ecl,max})$ relation, the time-scale, δt, needed to build-up a fully populated young star-cluster population can be determined as in § 3.4.1. The result is shown in Fig. 9. Note that in both the limited ($M_{\rm ecl,max*} = 10^7\,M_\odot$, Fig. 9) and the unlimited ($M_{\rm ecl,max*} = \infty$, Fig. 7) case it takes an arbitrarily long time to sample the CMF arbitrarily close to $M_{\rm ecl,max*}$.

4 DISCUSSION AND CONCLUSIONS

Observations of young star-cluster systems in disk galaxies show that there exists a correlation between the total SFR and the luminosity of the brightest star-cluster in the young-cluster population. This can be transformed to a SFR–heaviest-cluster-mass relation ($SFR(M_{\rm ecl,max})$, eq. 7).

Very young star-clusters in the MW and the LMC that are deduced to have formed within a few Myr follow a similar $SFR(M_{\rm ecl,max})$ relation, although this "local" relation is somewhat steeper if it is assumed that the formation time-scale of individual clusters is the same in all cases (≈ 1 Myr, Fig. 3). Taking instead the formation-time-scale to be the free-fall time of the pre-cluster molecular cloud core the correct slope is obtained if the pre-cluster cloud core radius is a few pc independent of cluster mass (Fig. 4). This implies that the cluster-forming molecular cloud cores may contract by a factor of 5 to 10 as the clusters form. That the pre-cluster radii appear to not vary much with cluster mass implies the pre-cluster cores to have increasing density with increasing mass. Indeed, Larsen (2003) finds young extra-galactic clusters to have only a mild increase of effective radius with mass, and embedded clusters from the local Milky Way also suggest the cluster radii to be approximately independent of cluster mass (Kroupa 2002; Kroupa & Boily 2002).

A model according to which the total mass of the young-cluster population, $M_{\rm tot}$, is assumed to be assembled in a star-formation "epoch" with an a-priori unknown duration, δt, gives the corresponding $SFR = M_{\rm tot}/\delta t$ and leads to good agreement with the empirical $SFR(M_{\rm ecl,max})$ relation for $1 \lesssim \delta t/{\rm Myr} \lesssim 10$. A particularly good match with the empirical relation results for $\delta t \approx$ few $\times\,10$ Myr and for a power-law CMF with $\beta \approx 2.35$. It should be noticed that the slope of this CMF for stellar clusters is virtually the same as for the Salpeter IMF ($\alpha = 2.35$) which applies for the early-type stars in these clusters. Conversely, adopting the empirical $SFR(M_{\rm ecl,max})$ relation, δt can be calculated for different young-cluster power-law mass functions with exponent β. We find that $\delta t \lesssim$ few $\times$ 10 Myr for $\beta \lesssim 2.4$. This value is nicely consistent with independent observations. For example, Hunter et al. (2003) find $2 < \beta < 2.4$ for a sample of 939 LMC and SMC clusters after applying corrections for redding, fading, evaporation and size-of-sample effects.

The same holds true if a fundamental maximum star-cluster mass near $M_{\rm ecl,max*} = 10^7\,M_\odot$ is introduced. The existence of such a fundamental maximum cluster mass is supported by "clusters" with $M \gtrsim 5 \times 10^6\,M_\odot$ having complex stellar populations more reminiscent of dwarf galaxies that cannot be the result of a truly single star-formation event.

The short time-span $\delta t \approx$ few $\times\,10$ Myr for completely-populating a CMF up to the maximum cluster mass of the population, $M_{\rm ecl,max} \leqslant M_{\rm ecl,max*}$, can be understood as being due to the high ambient pressures in the inter-stellar medium needed to raise the global SFR high enough for

populous star-clusters to be able to emerge. This short time-scale, which we refer to as a star-formation "epoch", does not preclude the star-formation activity in a galaxy to continue for many "epochs", whereby each epoch may well be characterised by different total young-star-cluster masses, $M_{\rm tot}$. According to this notion, dwarf galaxies may experience unfinished "epochs", in the sense that during the onset of an intense star-formation activity that may be triggered through a tidal perturbation for example, the ensuing feedback which may include galactic winds may momentarily squelch further star-formation within the dwarf such that the cluster system may not have sufficient time to completely populate the cluster mass function. Squelching would typically occur once the most massive cluster has formed. Dwarf galaxies would therefore deviate notably from the $M_{\rm ecl,max}(SFR)$ relation (§ 2).

The conclusion is therefore that the observed $SFR(M_{\rm ecl,max})$ data can be understood as being a natural outcome of star formation in clusters and that the SFR at a given epoch dictates the range of star-cluster masses formed given a CMF that appears to be a Salpeter power law. The associated formation time-scales are short being consistent with the conjecture by Elmegreen (2000) that star-formation is a very quick process on all scales. Within about 10^7 yr a complete cluster system is build (Fig. 6, § 3.3), while individual clusters form on a time scale of 10^6 years and stars only in about 10^5 years. Correspondingly, molecular-cloud life-times are short ($\approx$ few $\times$ 10 Myr) - suporting the assertion by Hartmann, Ballesteros-Paredes & Bergin (2001).

Applying the empirical $SFR(M_{\rm ecl,max})$ relation to the MW which has $SFR \approx 3 - 5\,M_\odot$/yr (Prantzos & Aubert 1995) a maximum cluster mass of about $10^5\,M_\odot$ is expected from eq. 6. It is interesting that only recently have Alves & Homeier (2003) revealed a very massive cluster in our Milky-way with about 100 O stars (similar to R136 in the LMC). Knödlseder (2000) notes that the Cygnus OB2 association contains 2600±400 OB stars and about 120 O stars with a total mass of $(4-10)\times 10^4\,M_\odot$, and that this "association" may be a very young globular-cluster-type object with a core radius of approximately 14 pc within the MW disk at a distance of about 1.6 kpc from the Sun (but see Bica, Bonatto & Dutra 2003). This object may be expanded after violent gas expulsion (Boily & Kroupa 2003a,b). The MW therefore does not appear to be unusual in its star-cluster production behaviour.

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